

# New pole contribution to $P_{h\perp}$ -weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering

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## Abstract

In this paper, we discuss the new hard pole contribution to the  $P_{h\perp}$ -weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering. We perform the complete next-to-leading order calculation of the  $P_{h\perp}$ -weighted cross section and show that the new hard pole contribution is required in order to obtain the complete evolution equation for the Qiu-Sterman function derived by different approaches.

# 1 Introduction

The origin of the single transverse-spin asymmetries(SSAs) in various hard processes has been a longstanding problem for almost 40 years since the unexpected large asymmetries were observed in mid-1970s [1, 2]. Many theoretical works in recent decades found that twist-3 framework in collinear factorization approach is a possible extended framework which can provide a systematic description of the large SSA in perturbative QCD. The twist-3 framework has been well developed in leading-order(LO) accuracy [3-17] in recent decades. Started with the pioneering work by Efremov and Teryaev [3], the more systematic calculation was presented by Qiu and Sterman [4-6]. While the formalism was applied to SSAs in other processes [7, 8, 9], the solid foundation was finally provided in [11] to provide the gauge-invariant twist-3 cross section formula in terms of the complete set of the twist-3 distribution functions. The phenomenological analysis [10, 18] showed that the twist-3 distribution effect of the transversely polarized proton can give a reasonable description of the experimental data and therefore it is widely believed that this effect is one of possible sources of the large SSA.

In usual perturbative QCD calculation, higher-order corrections are often not negligible compared to a leading order contribution. Those corrections bring the logarithmic energy-scale dependence of nonperturbative function which is described by the evolution equation. Systematic treatment of the scale dependence of the twist-3 functions is essential to a quantitative description of the SSA. The twist-3 distribution effect of the transversely polarized proton is embodied as the so-called Qiu-Sterman (QS) function in the spin-dependent cross section formula. The scale evolution equation of the QS function was discussed by using several different approaches so far [19-26]. One of the approaches is the next-to-leading-order (NLO) calculation of the transverse momentum  $P_{h\perp}$ -weighted cross section. Based on this approach, some part of the evolution equation was first derived in the study of the Drell-Yan process [21]. Subsequently the authors of [26] examined the so-called hard-pole (HP) contribution in the semi-inclusive deep inelastic scattering (SIDIS) and identified an extra term in the evolution equation which had been derived by other approaches [22, 23, 24, 25], while the complete agreement for the whole evolution equation was not yet achieved. In the meanwhile the authors of [27] found the new HP contribution in the study of the  $P_{h\perp}$ -differential cross section for SSA in SIDIS. In this paper, we include this new HP contribution for the NLO  $P_h$ -weighted cross section. We shall show that this new HP contribution yields extra collinear singularity and its factorization reproduce the correct evolution of the QS function found in [22, 24, 25]. We shall also present the complete NLO cross section for the twist-3  $P_h$ -weighted cross section for SSA.

The remainder of the paper is organized as follows: in Sec. 2 we introduce the twist-3 distribution functions for the transversely polarized proton. Next, in Sec. 3 we discuss the contribution of the real-emission diagrams in NLO  $P_{h\perp}$ -weighted cross section. In Sec. 4 we introduce the LO and NLO virtual-correction contributions which were already calculated in the previous work and present the complete NLO cross section formula. Finally, in Sec. 5 we summarize our work.

## 2 Twist-3 distribution functions for transversely polarized proton

Here we introduce twist-3 functions relevant to our study. The F-type twist-3 functions are defined as

$$M_{Fij}^\alpha(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp | \bar{\psi}_j(0) g F^{\alpha n}(\mu n) \psi_i(\lambda n) | pS_\perp \rangle$$

$$= \frac{M_N}{4} \epsilon^{\alpha p n S_\perp} (\not{p})_{ij} G_F(x_1, x_2) + i \frac{M_N}{4} S_\perp^\alpha (\gamma_5 \not{p})_{ij} \tilde{G}_F(x_1, x_2) \cdots, \quad (1)$$

where  $F^{\alpha n}$  is a gluon's field strength tensor and we used the simplified notation  $F^{\alpha\beta} n_\beta$  and  $\epsilon^{\alpha\beta p n} \equiv \epsilon^{\alpha\beta\rho\sigma} p_\rho n_\sigma$ . The anti-symmetric tensor is defined as  $\epsilon^{0123} = -1$ . We introduced the nucleon mass  $M_N$  in order to define the dimensionless functions. From the Hermiticity and  $PT$ -invariance, one can show the following symmetry properties:

$$G_F(x_1, x_2) = G_F(x_2, x_1), \quad \tilde{G}_F(x_1, x_2) = -\tilde{G}_F(x_2, x_1). \quad (2)$$

In this paper, we discuss the evolution equation of the QS function  $G_F(x_1, x_2)$  at  $x_1 = x_2$ .

### 3 Contribution of real-emission diagrams to next-leading order cross section

We consider the SSA for light-hadron production in SIDIS,

$$e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + h(P_h) + X. \quad (3)$$

Within the collinear factorization framework, the SSA can be described by the twist-3 effects. In this process, the SSA receives two types of twist-3 contributions, the distribution effect of the transversely polarized proton and the fragmentation effect of the light-hadron. We focus on the former contribution in this study to derive the evolution equation of the Qiu-Sterman function  $G_F(x, x)$ . In the case of SIDIS, the cross section formula can be expressed in terms of the following Lorentz invariant variables,

$$S_{ep} = (p + \ell)^2, \quad Q^2 = -q^2, \quad x_B = \frac{Q^2}{2p \cdot q}, \quad z_h = \frac{p \cdot P_h}{p \cdot q}, \quad (4)$$

where  $q = (\ell - \ell')$  is the momentum of the virtual photon. We choose the hadron frame [17] for the calculation,

$$\ell = \frac{Q}{2} (\cosh \psi, \sinh \psi \cos \phi, \sinh \psi \sin \phi, -1), \quad (5)$$

$$q = (0, 0, 0, -Q), \quad p^\mu = \left( \frac{Q}{2x_B}, 0, 0, \frac{Q}{2x_B} \right), \quad S_\perp^\mu = (0, \cos \Phi_S, \sin \Phi_S, 0) \quad (6)$$

$$P_h = \frac{z_h Q}{2} \left( 1 + \frac{P_{h\perp}^2}{z_h^2 Q^2}, \frac{2P_{h\perp}}{z_h Q} \cos \chi, \frac{2P_{h\perp}}{z_h Q} \sin \chi, \frac{P_{h\perp}^2}{z_h^2 Q^2} - 1 \right), \quad (7)$$

where  $\cosh \psi = \frac{2x_B S_{ep}}{Q^2} - 1$ . In this paper, we discuss the NLO  $P_{h\perp}$ -weighted polarized cross section defined as

$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle}{dx_B dQ^2 dz_h d\phi} \equiv \int d^2 P_{h\perp} \epsilon^{S_\perp P_{h\perp} p n} \left( \frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_h dP_{h\perp}^2 d\phi d\chi} \right). \quad (8)$$

First we consider the real-emission diagrams in NLO contribution. The NLO real-emission diagrams in  $P_{h\perp}$ -weighted cross section are the same as the LO diagrams in  $P_{h\perp}$ -differential

case [11, 27]. The calculation technique to derive a twist-3 cross section for  $2 \rightarrow 2$  scattering has been well developed in recent decades and a systematic way to derive the gauge-invariant cross section was established in [11]. We briefly discuss the derivation below. The cross section for SIDIS was presented in [17, 28] as

$$\frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_h dP_{h\perp}^2 d\phi d\chi} = \frac{\alpha_{em}^2}{128\pi^4 z_h x_B^2 S_{ep}^2 Q^2} L_{\mu\nu} W^{\mu\nu}. \quad (9)$$

where  $\alpha_{em} = \frac{e^2}{4\pi}$  is the QED coupling constant and  $L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu) - Q^2 g_{\mu\nu}$  is the leptonic tensor. Since we are interested in the twist-3 effect of the transversely polarized proton, we introduce the usual twist-2 fragmentation function  $D(z)$  for fragmentation part as

$$W^{\mu\nu} = \int \frac{dz}{z^2} D(z) w^{\mu\nu} \quad (10)$$

The hadronic tensor  $w^{\mu\nu}$  describes a scattering of the virtual photon and the transversely polarized proton. We consider a “general” diagram given by

$$\begin{aligned} w^{\mu\nu} = & \int d^4\xi \int d^4\eta \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i\eta \cdot (k_2 - k_1)} \langle PS_\perp | \bar{\psi}_j(0) g A_\alpha(\eta) \psi_i(\xi) | PS_\perp \rangle \\ & \times \left( S_{ji}^\alpha(k_1, k_2) + \tilde{S}_{ji}^\alpha(k_1, k_2) \right), \end{aligned} \quad (11)$$

which represents the scattering of the virtual photon and the polarized proton graphically shown in Fig.1. We suppressed the Lorentz indices  $\mu$  and  $\nu$  of the hard parts  $S_{ji}^\alpha(k_1, k_2)$  and  $\tilde{S}_{ji}^\alpha(k_1, k_2)$  for simplicity. Within the collinear factorization framework, a complex phase required for the naively  $T$ -odd SSA can be provided by a pole contribution associated with a internal propagator. In SIDIS case, the pole contributions can be classified into four types as soft-gluon-pole(SGP), soft-fermion-pole(SFP), hard-pole(HP) and another hard-pole(HP2) which are respectively shown in Fig. 2-5. We would like to emphasize that the HP2 contribution was not considered in previous studies for the  $P_{h\perp}$ -weighted cross section and this contribution is essential to obtain the consistent evolution equation of  $G_F(x_1, x_2)$  with the results in different approaches [22, 24, 25]. We can check that the hard part  $S_{ji}^\alpha(k_1, k_2)$  with the pole contribution satisfies the Ward identity

$$(k_2 - k_1)_\alpha S_{ji}^{\text{pole } \alpha}(k_1, k_2) = 0, \quad (12)$$

and associated relations

$$(x_2 - x_1) \frac{\partial}{\partial k_2^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=x_i p} = -S_{ji}^{\text{pole } \alpha}(x_1 p, x_2 p), \quad (13)$$

$$(x_2 - x_1) \frac{\partial}{\partial k_1^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=x_i p} = S_{ji}^{\text{pole } \alpha}(x_1 p, x_2 p). \quad (14)$$

For SFP and HP contributions, the above relations give

$$\frac{\partial}{\partial k_2^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=x_i p} = -\frac{\partial}{\partial k_1^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=x_i p}, \quad (15)$$

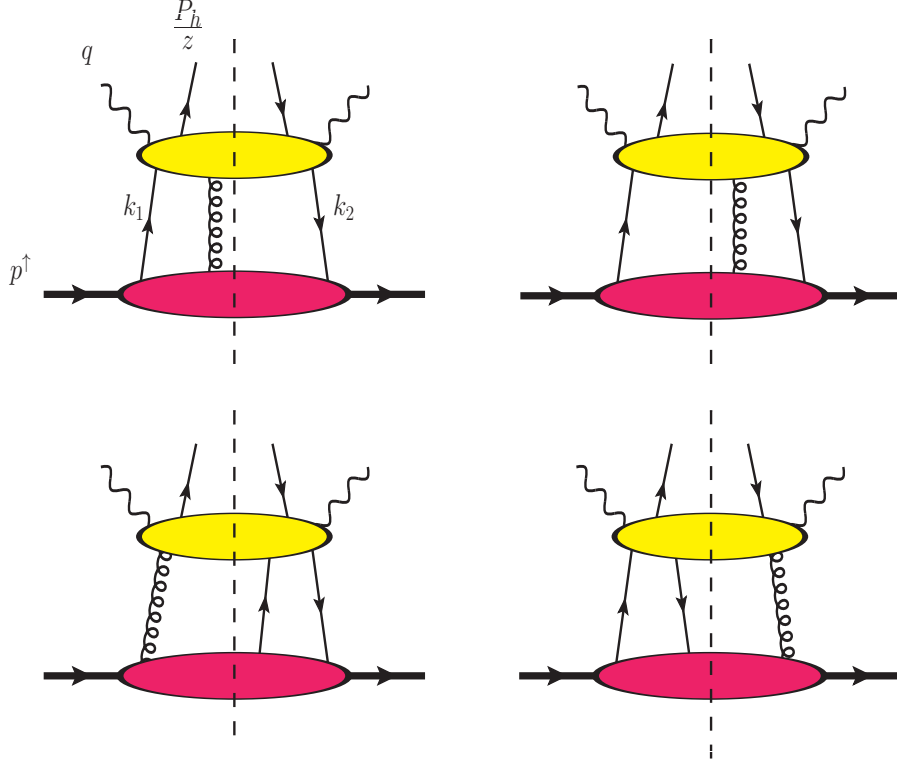


Figure 1: Diagrammatic description for the hadronic tensor  $w^{\mu\nu}$ . The upper diagrams and the lower diagrams respectively represent  $S_{ji}^\alpha(k_1, k_2)$  and  $\tilde{S}_{ji}^\alpha(k_1, k_2)$ .

and we can find the same relation for SGP contribution with a direct inspection. Another hard part  $\tilde{S}_{ji}^{\text{pole } p}(k_1, k_2)$  also has the same relations. To extract the twist-3  $O(k_\perp)$  contribution from the general contribution (11), we perform the collinear expansion for the hard parts as

$$\begin{aligned}
S_{ji}^{\text{pole } \alpha}(k_1, k_2) &= S_{ji}^{\text{pole } \alpha}((k_1 \cdot n)p, (k_2 \cdot n)p) + \frac{\partial}{\partial k_1^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=(k_i \cdot n)p} \omega_\beta^\alpha k_1^\beta \\
&\quad + \frac{\partial}{\partial k_2^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=(k_i \cdot n)p} \omega_\beta^\alpha k_2^\beta \\
&= S_{ji}^{\text{pole } \alpha}((k_1 \cdot n)p, (k_2 \cdot n)p) + \frac{\partial}{\partial k_2^\alpha} S_{ji}^{\text{pole } p}(k_1, k_2) \Big|_{k_i=(k_i \cdot n)p} \omega_\beta^\alpha (k_2^\beta - k_1^\beta), \quad (16)
\end{aligned}$$

where  $\omega_\beta^\alpha = g_\beta^\alpha - p^\alpha n_\beta$  and we used the relation (15). And we separate the Lorentz components of the gluon field,

$$A^\alpha = A^n p^\alpha + \omega_\beta^\alpha A^\beta. \quad (17)$$

Then we pick up subleading contributions in (11) and construct the F-type correlator (1) as

$$w^{\mu\nu} = \int d^4\xi \int d^4\eta \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i\eta \cdot (k_2 - k_1)} \langle PS_\perp | \bar{\psi}_j(0) g A^n(\eta) \psi_i(\xi) | PS_\perp \rangle$$

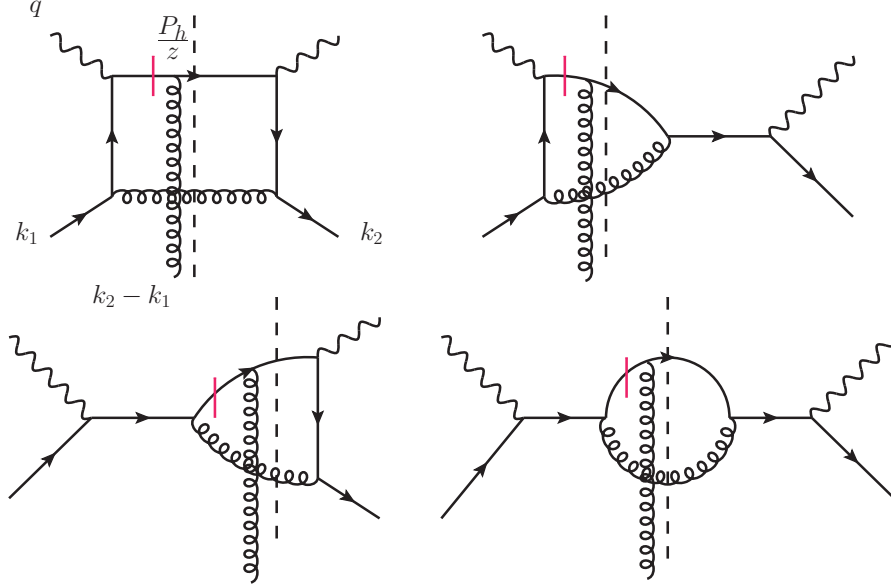


Figure 2: Diagrammatic description for SGP diagrams  $H_{Lji}^{\text{SGP}\alpha}(k_1, k_2)$ . Barred propagators provide the pole contribution.

$$\begin{aligned}
& \times \frac{\partial}{\partial k_2^\alpha} \left( S_{ji}^{\text{pole } p}(k_1, k_2) + \tilde{S}_{ji}^{\text{pole } p}(k_1, k_2) \right) \Big|_{k_i=(k_i \cdot n)p} \omega_\beta^\alpha (k_2^\beta - k_1^\beta) \\
& + \int d^4 \xi \int d^4 \eta \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i\eta \cdot (k_2 - k_1)} \langle PS_\perp | \bar{\psi}_j(0) g \omega_\alpha^\beta A_\beta(\eta) \psi_i(\xi) | PS_\perp \rangle \\
& \times \left( S_{ji}^{\text{pole } p}((k_1 \cdot n)p, (k_2 \cdot n)p) + \tilde{S}_{ji}^{\text{pole } p}((k_1 \cdot n)p, (k_2 \cdot n)p) \right) \\
& = i \omega_\alpha^\beta \int dx_1 \int dx_2 M_{ijF}^\alpha(x_1, x_2) \frac{\partial}{\partial k_2^\beta} \left( S_{ji}^{\text{pole } p}(k_1, k_2) + \tilde{S}_{ji}^{\text{pole } p}(k_1, k_2) \right) \Big|_{k_i=x_i p} \quad (18)
\end{aligned}$$

We express the hard parts in terms of each pole contribution as

$$\begin{aligned}
S_{ji}^{\text{pole}\alpha}(k_1, k_2) &= H_{Lji}^{\text{SGP}\alpha}(k_1, k_2) \left\{ -i\pi \delta\left(\left(\frac{P_h}{z} - (k_2 - k_1)\right)^2\right) \right\} (2\pi) \delta\left((k_2 + q - \frac{P_h}{z})^2\right) \\
&+ H_{Lji}^{\text{HP}\alpha}(k_1, k_2) \left\{ -i\pi \delta\left((k_1 + q)^2\right) \right\} (2\pi) \delta\left((k_2 + q - \frac{P_h}{z})^2\right) \\
&+ H_{Lji}^{\text{SFP}\alpha}(k_1, k_2) \left\{ -i\pi \delta\left(\left(\frac{P_h}{z} - (k_2 - k_1) - q\right)^2\right) \right\} (2\pi) \delta\left((k_2 + q - \frac{P_h}{z})^2\right) \\
&+ \text{mirror diagrams} \quad (19)
\end{aligned}$$

$$\tilde{S}_{ji}^{\text{pole}\alpha}(k_1, k_2) = \tilde{H}_{Lji}^{\text{HP}2\alpha}(k_1, k_2) \left\{ i\pi \delta\left((k_2 + q)^2\right) \right\} (2\pi) \delta\left((k_2 - k_1 + q - \frac{P_h}{z})^2\right)$$

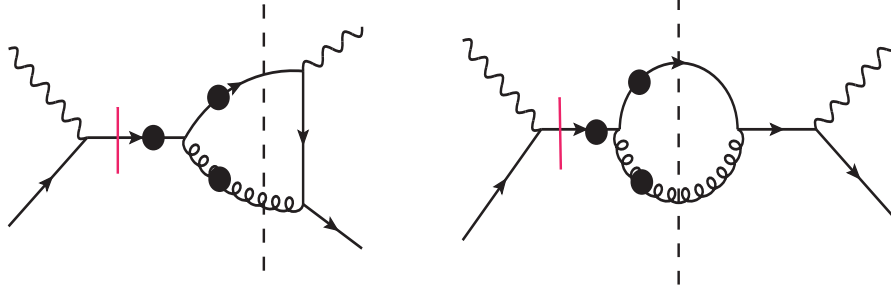


Figure 3: Diagrammatic description for HP diagrams  $H_{Lji}^{\text{HP}\alpha}(k_1, k_2)$ . The third gluon line with momentum  $k_2 - k_1$  which comes from the transversely polarized proton attaches to one of the black dots in each diagram.

$$\begin{aligned}
& +\tilde{H}_{Lji}^{\text{SFP}\alpha}(k_1, k_2) \left\{ i\pi\delta\left(\left(\frac{P_h}{z} - k_2 - q\right)^2\right) \right\} (2\pi)\delta\left((k_2 - k_1 + q - \frac{P_h}{z})^2\right) \\
& +\text{mirror diagrams}
\end{aligned} \tag{20}$$

We can find that the SFP contributions  $H_{ji}^{\text{SFP}p}(k_1, k_2)$  and  $\tilde{H}_{ji}^{\text{SFP}p}(k_1, k_2)$  are the topologically same and then exactly cancel each other. After a little computation, we can obtain the formula for the hadronic tensor  $W^{\mu\nu}$  as follows.

$$\begin{aligned}
W^{\mu\nu} = & \frac{M_N\pi^2}{2} \int \frac{dz}{z^2} D(z) \int \frac{dx}{x} \delta\left((xp + q - \frac{P_h}{z})^2\right) \left[ -2\epsilon^{p_c p n S_\perp} \frac{d}{dx} G_F(x, x) \frac{\hat{s} + Q^2}{\hat{t}\hat{u}} \text{Tr}[x\not{p}H(xp)] \right. \\
& - 2\epsilon^{p_c p n S_\perp} G_F(x, x) \frac{\hat{s} + Q^2}{\hat{t}\hat{u}} \left\{ Q^2 \left( \frac{\partial}{\partial \hat{s}} - \frac{\partial}{\partial Q^2} \right) \text{Tr}[x\not{p}H(xp)] \right\} \\
& + G_F(x, x_B) \frac{1}{\hat{x} - 1} \frac{\hat{x}}{Q^2} \epsilon_\alpha^{p n S_\perp} \left( \text{Tr}[x\not{p}H_L^{\text{HP}\alpha}(x_B p, xp)] + \text{Tr}[x\not{p}H_R^{\text{HP}\alpha}(xp, x_B p)] \right) \\
& - \tilde{G}_F(x, x_B) \frac{1}{\hat{x} - 1} \frac{\hat{x}}{Q^2} iS_{\perp\alpha} \left( \text{Tr}[\gamma_5 x\not{p}\tilde{H}_L^{\text{HP}\alpha}(x_B p, xp)] - \text{Tr}[\gamma_5 x\not{p}\tilde{H}_R^{\text{HP}\alpha}(xp, x_B p)] \right) \\
& + G_F(x_B, x_B - x) \frac{\hat{x}}{Q^2} \epsilon_\alpha^{p n S_\perp} \left( \text{Tr}[x\not{p}H_L^{\text{HP}2\alpha}((x_B - x)p, x_B p)] \right. \\
& \left. + \text{Tr}[x\not{p}H_R^{\text{HP}2\alpha}(x_B p, (x_B - x)p)] \right) \\
& - \tilde{G}_F(x_B, x_B - x) \frac{\hat{x}}{Q^2} iS_{\perp\alpha} \left( \text{Tr}[\gamma_5 x\not{p}\tilde{H}_L^{\text{HP}2\alpha}((x_B - x)p, x_B p)] \right. \\
& \left. - \text{Tr}[\gamma_5 x\not{p}\tilde{H}_R^{\text{HP}2\alpha}(x_B p, (x_B - x)p)] \right),
\end{aligned} \tag{21}$$

where we used the Mandelstam variables

$$\hat{s} = (xp + q)^2 = \frac{1 - \hat{x}}{\hat{x}} Q^2, \tag{22}$$

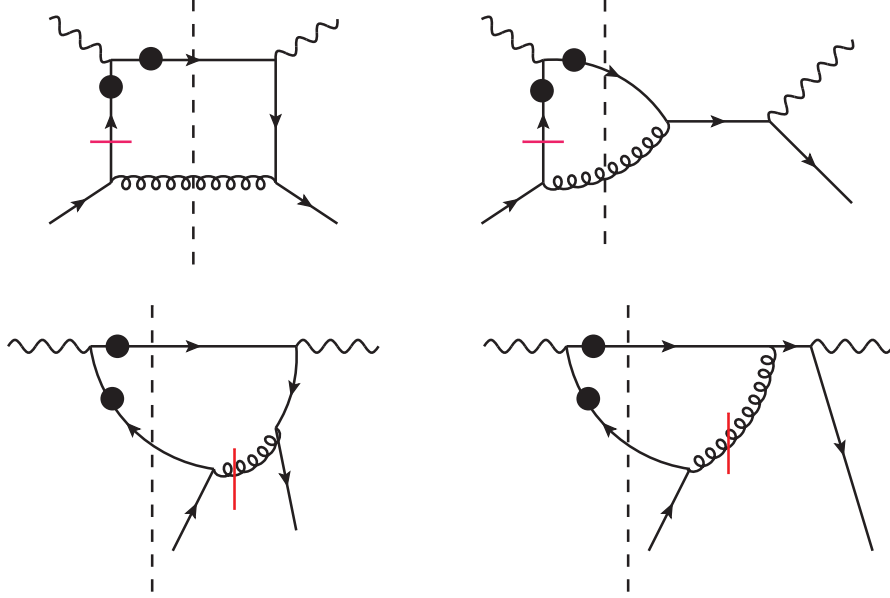


Figure 4: Diagrammatic description for SFP diagrams. The upper diagrams and the lower diagrams respectively represent  $H_{ji}^{\text{SFP}\alpha}(k_1, k_2)$  and  $\tilde{H}_{ji}^{\text{SFP}\alpha}(k_1, k_2)$ .

$$\hat{t} = (p_c - q)^2 = -\frac{1 - \hat{z}}{\hat{x}} Q^2, \quad (23)$$

$$\hat{u} = (xp - p_c)^2 = -\frac{\hat{z}}{\hat{x}} Q^2, \quad (24)$$

where  $p_c = \frac{P_h}{z}$ . We used the Ward identity (13) for hard-pole contributions. For the SGP contribution, we used master formula [28, 29]

$$\begin{aligned} \frac{\partial}{\partial k_2^\beta} \text{Tr}[x_1 \not{p} S_{ji}^{\text{SGP}p}(k_1, k_2)] \Big|_{k_i=x_i p} &= -i\pi\delta(x_1 - x_2) \frac{d}{dp_c^\beta} \text{Tr}[x_1 \not{p} S(x_1 p)] \\ &= 2i\pi\delta(x_1 - x_2) \frac{\hat{s} + Q^2}{\hat{u}} p_c^\beta \frac{\partial}{\partial \hat{t}} \text{Tr}[x_1 \not{p} S(x_1 p)], \end{aligned} \quad (25)$$

where  $S(xp)$  is the  $2 \rightarrow 2$  scattering cross section without the third gluon line comes from the transversely polarized proton (but the color factor is the same as  $S_{ji}^{\text{SGP}p}$ ). In this paper, we consider the metric contribution,

$$L_{\mu\nu} W^{\mu\nu} \rightarrow (-g_{\mu\nu} W^{\mu\nu}), \quad (26)$$

$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{real}}}{dx_B dQ^2 dz_h d\phi} = \frac{\alpha_{em}^2}{32\pi^2 z_h x_B^2 S_{ep}^2 Q^2} \int dz z D(z) \int \frac{d^2 p_{c\perp}}{(2\pi)^2} \epsilon^{S_{\perp p_{c\perp} p n}} (-g_{\mu\nu} w^{\mu\nu}), \quad (27)$$

and the metric should be normalized as  $g_{\mu\nu} \rightarrow \frac{1}{1-\epsilon} g_{\mu\nu}$  with  $\epsilon = 2 - D/2$  in  $D$ -dimensional calculation. We can compute the  $P_{h\perp}$ -weighted cross section for NLO real-emission diagrams in



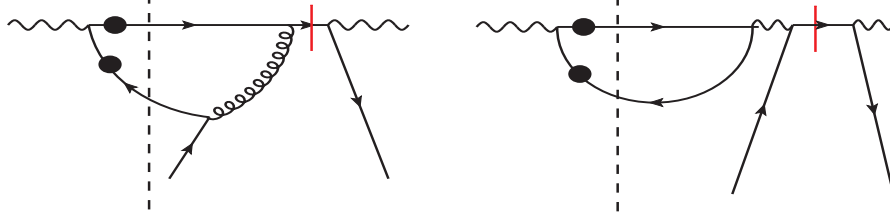


Figure 5: Diagrammatic description for HP2 diagrams  $H_{Lji}^{\text{HP}2\alpha}(k_1, k_2)$ . These diagrams were first found in [27] in the study of  $P_{h\perp}$ -differential SSA but was not considered in previous studies of the  $P_{h\perp}$ -weighted SSA.

$D$ -dimension as follows.

$$\begin{aligned}
& \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{real}}}{dx_B dQ^2 dz_h d\phi} \\
&= -\frac{\pi M_N \alpha_{em}^2 \alpha_s}{2x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 \int dz D^q(z) \mu^{2\epsilon} \int \frac{d^{2-2\epsilon} p_{c\perp}}{(2\pi)^{2-2\epsilon}} \left[ \int \frac{dx}{x} \delta\left(p_{c\perp}^2 - \frac{(1-\hat{x})(1-\hat{z})\hat{z}}{\hat{x}} Q^2\right) \right. \\
&\quad \times \frac{1}{1-\epsilon} \left[ \frac{d}{dx} G_F^q(x, x) H_D + G_F^q(x, x) H_{ND} + G_F^q(x, x_B) H_{HP} + \tilde{G}_F^q(x, x_B) H_{HPT} \right. \\
&\quad \left. \left. + G_F^q(x_B, x_B - x) H_{HP2} + \tilde{G}_F^q(x_B, x_B - x) H_{HPT2} \right] \right], \tag{28}
\end{aligned}$$

where  $q$  denotes the quark flavor,  $\alpha_s$  is the QCD coupling constant and we used the symmetry for  $p_{c\perp}$ -integral

$$\begin{aligned}
\int d^{2-2\epsilon} p_{c\perp} p_{c\perp\alpha} p_{c\perp\beta} \epsilon^{S_\perp \alpha p n} \epsilon^{\beta p n S_\perp} &= - \int d^{2-2\epsilon} p_{c\perp} \frac{1}{2(1-\epsilon)} p_{c\perp}^2 g_{\perp\alpha\beta} \epsilon^{S_\perp \alpha p n} \epsilon^{\beta p n S_\perp} \\
&= - \int d^{2-2\epsilon} p_{c\perp} \frac{1}{2(1-\epsilon)} \frac{(1-\hat{x})(1-\hat{z})\hat{z}}{\hat{x}} Q^2, \tag{29}
\end{aligned}$$

and the hard cross sections can be computed as

$$H_D = \frac{1}{2N} \left\{ 1 - 2\hat{x} - \hat{z} + \epsilon(1 - 2\hat{x} + \hat{z}) + \frac{1 + \hat{x}^2 - \epsilon(1 - \hat{x})^2}{1 - \hat{z}} \right\} \tag{30}$$

$$\begin{aligned}
H_{ND} &= \frac{1}{2N} \left[ -\frac{2}{(1-\hat{x})(1-\hat{z})} + \frac{1 + \hat{z} + \epsilon(1-\hat{z})}{1-\hat{x}} \right. \\
&\quad \left. + \frac{(1-\hat{x})(1+2\hat{x}) - \epsilon(1-\hat{x})(2\hat{x}-1)}{1-\hat{z}} - 2(1+\epsilon)(1-\hat{x}) \right] \tag{31}
\end{aligned}$$

$$H_{HP} = \left( \hat{z} C_F + \frac{1}{2N} \right) \left[ \frac{2}{(1-\hat{x})(1-\hat{z})} - \frac{1 + \hat{z} + \epsilon(1-\hat{z})}{1-\hat{x}} - \frac{1}{1-\hat{z}} + (1 + \hat{z} + \epsilon) \right] \tag{32}$$

$$H_{HPT} = \frac{1}{1-\epsilon} \left( \hat{z} C_F + \frac{1}{2N} \right) \left[ -\frac{1+\hat{z}-2\epsilon+\epsilon^2(1-\hat{z})}{1-\hat{x}} - \frac{1+\epsilon}{1-\hat{z}} + (1+\hat{z}+\epsilon\hat{z}+\epsilon^2) \right] \quad (33)$$

$$H_{HP2} = \frac{1}{2N} \left[ \frac{1-2\hat{x}}{1-\hat{z}} - (1-2\hat{x})(1+\hat{z}+\epsilon) \right] + \frac{1}{1-\epsilon} \frac{1}{2} \left[ (1-2\hat{x})(2\hat{z}^2-2\hat{z}+1-\epsilon) \right] \quad (34)$$

$$H_{HPT2} = \frac{1}{1-\epsilon} \frac{1}{2N} \left[ \frac{1+\epsilon}{1-\hat{z}} - (1+\hat{z}+\epsilon\hat{z}+\epsilon^2) \right] - \frac{1}{1-\epsilon} \frac{1}{2} \left[ 1-2\hat{z}-\epsilon \right], \quad (35)$$

where  $N = 3$  is a number of colors and  $C_F = \frac{N^2-1}{2N}$ . The  $p_{c\perp}$ -integral can be calculated in  $D$ -dimension as

$$\begin{aligned} & \int \frac{d^{2-2\epsilon} p_c}{(2\pi)^{2-2\epsilon}} \delta \left( p_{c\perp}^2 - \frac{(1-\hat{x})(1-\hat{z})\hat{z}}{\hat{x}} Q^2 \right) \\ &= \frac{1}{(2\pi)^{2-2\epsilon}} \int dp_{c\perp} \int d\Omega_{2-2\epsilon} (p_{c\perp})^{1-2\epsilon} \delta \left( p_{c\perp}^2 - \frac{(1-\hat{x})(1-\hat{z})\hat{z}}{\hat{x}} Q^2 \right) \\ &= \frac{1}{4\pi} \left( \frac{4\pi}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left( \frac{(1-\hat{x})(1-\hat{z})\hat{z}}{\hat{x}} \right)^{-\epsilon}, \end{aligned} \quad (36)$$

where  $\Omega_{2-2\epsilon}$  is a solid angle

$$\int d\Omega_{2-2\epsilon} = \frac{2\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}. \quad (37)$$

We carry out the  $\epsilon$ -expansion for the phase-space integral as follows.

$$\hat{z}^{-\epsilon} \simeq 1 - \epsilon \ln \hat{z}, \quad \hat{x}^\epsilon \simeq 1 + \epsilon \ln \hat{z}, \quad (38)$$

$$(1-\hat{z})^{-1-\epsilon} \simeq -\frac{1}{\epsilon} \delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+, \quad (39)$$

$$(1-\hat{x})^{-1-\epsilon} \simeq -\frac{1}{\epsilon} \delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+, \quad (40)$$

Then the cross section formula reads

$$\begin{aligned} & \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{real}}}{dx_B dQ^2 dz_h d\phi} \\ &= -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \frac{\alpha_s}{2\pi} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \sum_q e_q^2 \left[ \int dz D^q(z) \int \frac{dx}{x} \left[ \frac{d}{dx} G_F^q(x, x) \hat{\sigma}_D + G_F^q(x, x) \hat{\sigma}_{ND} \right. \right. \\ & \quad \left. \left. + G_F^q(x, x_B) \hat{\sigma}_{HP} + \tilde{G}_F^q(x, x_B) \hat{\sigma}_{HPT} + G_F^q(x_B, x_B - x) \hat{\sigma}_{HP2} + \tilde{G}_F^q(x_B, x_B - x) \hat{\sigma}_{HPT2} \right] \right], \end{aligned} \quad (41)$$

$$\hat{\sigma}_D = \frac{1}{2N} \left[ \left( -\frac{1}{\epsilon} \right) (1+\hat{x}^2) \delta(1-\hat{z}) + (1-\hat{z}) + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} \right]$$

$$-\delta(1-\hat{z})\left((1+\hat{x}^2)\ln\frac{\hat{x}}{1-\hat{x}}+2\hat{x}\right)] \quad (42)$$

$$\begin{aligned} \hat{\sigma}_{ND} = & \frac{1}{2N}\left[(-\frac{2}{\epsilon^2})\delta(1-\hat{x})\delta(1-\hat{z})+(-\frac{1}{\epsilon})\left(2\delta(1-\hat{x})\delta(1-\hat{z})-\frac{1+\hat{z}^2}{(1-\hat{z})_+}\delta(1-\hat{x})\right.\right. \\ & +\frac{2\hat{x}^3-3\hat{x}^2-1}{(1-\hat{x})_+}\delta(1-\hat{z}))\left.-2\delta(1-\hat{x})\delta(1-\hat{z})+\frac{2\hat{x}^3-3\hat{x}^2-1}{(1-\hat{x})_+(1-\hat{z})_+}\right. \\ & +\frac{1+\hat{z}}{(1-\hat{x})_+}-2(1-\hat{x})+\delta(1-\hat{z})\left(-(1-\hat{x})(1+2\hat{x})\log\frac{\hat{x}}{1-\hat{x}}-2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+\right. \\ & +\frac{2}{(1-\hat{x})_+}-2(1-\hat{x})+2\frac{\ln\hat{x}}{(1-\hat{x})_+}\left.)+\delta(1-\hat{x})\left((1+\hat{z})\ln\hat{z}(1-\hat{z})-2\frac{\ln\hat{z}}{(1-\hat{z})_+}\right.\right. \\ & \left.\left.-2\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_++\frac{2\hat{z}}{(1-\hat{z})_+}\right)\right] \quad (43) \end{aligned}$$

$$\begin{aligned} \hat{\sigma}_{HP} = & \left(\hat{z}C_F+\frac{1}{2N}\right)\left[\frac{2}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z})+\frac{1}{\epsilon}\left(2\delta(1-\hat{x})\delta(1-\hat{z})-\frac{1+\hat{z}^2}{(1-\hat{z})_+}\delta(1-\hat{x})\right.\right. \\ & \left.-\frac{1+\hat{x}}{(1-\hat{x})_+}\delta(1-\hat{z})\right)+2\delta(1-\hat{x})\delta(1-\hat{z})+\frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} \\ & +\delta(1-\hat{z})\left(\log\frac{\hat{x}}{1-\hat{x}}+2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+-2\frac{\ln\hat{x}}{(1-\hat{x})_+}-\frac{1+\hat{x}}{(1-\hat{x})_+}\right) \\ & +\delta(1-\hat{x})\left(-(1+\hat{z})\ln\hat{z}(1-\hat{z})+2\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_++2\frac{\ln\hat{z}}{(1-\hat{z})_+}-\frac{2\hat{z}}{(1-\hat{z})_+}\right)\left] \quad (44) \end{aligned}$$

$$\hat{\sigma}_{HPT} = \left(\hat{z}C_F+\frac{1}{2N}\right)\left[\frac{1}{\epsilon}\delta(1-\hat{z})-\frac{1-\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+}+\delta(1-\hat{z})\left(\ln\frac{\hat{x}}{1-\hat{x}}+3\right)\right] \quad (45)$$

$$\begin{aligned} \hat{\sigma}_{HP2} = & \frac{1}{2N}\left[-\frac{1}{\epsilon}(1-2\hat{x})\delta(1-\hat{z})+\frac{(1-2\hat{x})\hat{z}^2}{(1-\hat{z})_+}-\delta(1-\hat{z})(1-2\hat{x})\left(\ln\frac{\hat{x}}{1-\hat{x}}+1\right)\right] \\ & +\frac{1}{2}(1-2\hat{x})((1-\hat{z})^2+\hat{z}^2) \quad (46) \end{aligned}$$

$$\hat{\sigma}_{HPT2} = \frac{1}{2N}\left[-\frac{1}{\epsilon}\delta(1-\hat{z})+\frac{\hat{z}^2}{(1-\hat{z})_+}-\delta(1-\hat{z})\left(\ln\frac{\hat{x}}{1-\hat{x}}+3\right)\right]-\frac{1}{2}(1-2\hat{z}), \quad (47)$$

where we used the antisymmetric property  $\tilde{G}_F(x, x_B)\delta(1-\hat{x}) = \tilde{G}_F(x, x)\delta(1-\hat{x}) = 0$ . Finally we can derive the contribution of real-emission diagrams as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\text{real}}}{dx_B dQ^2 dz_h d\phi}$$

$$\begin{aligned}
&= -\frac{z_h \pi M_N \alpha_{em}^2 \alpha_s}{4x_B^2 S_{ep}^2 Q^2} \frac{1}{2\pi} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \sum_q e_q^2 \left[ C_F \frac{2}{\epsilon^2} G_F^q(x_B, x_B) D^q(z_h) \right. \\
&\quad + \left( -\frac{1}{\epsilon} \right) \left\{ D^q(z_h) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ C_F \frac{1+\hat{x}^2}{(1-\hat{x})_+} G_F^q(x, x) + \frac{N}{2} \left( \frac{(1+\hat{x})G_F^q(x_B, x) - (1+\hat{x}^2)G_F^q(x, x)}{(1-\hat{x})_+} \right. \right. \right. \right. \\
&\quad \left. \left. \left. + \tilde{G}_F^q(x_B, x) \right] \right\} - N G_F^q(x_B, x_B) + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1-2\hat{x})G_F^q(x_B, x_B-x) + \tilde{G}_F^q(x_B, x_B-x) \right) \right\} \\
&\quad + G_F^q(x_B, x_B) C_F \int_{z_h}^1 \frac{dz}{z} \frac{1+\hat{z}^2}{(1-\hat{z})_+} D^q(z) \left\{ \right. \\
&\quad + \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} \left\{ x \frac{dx}{x} G_F^q(x, x) D^q(z) \frac{1}{2N\hat{z}} \left[ 1 - \hat{z} + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} \right. \right. \\
&\quad \left. \left. - \delta(1-\hat{z}) \left( (1+\hat{x}^2) \ln \frac{\hat{x}}{1-\hat{x}} + 2\hat{x} \right) \right] + G_F^q(x, x) D^q(z) \frac{1}{2N\hat{z}} \left[ -2\delta(1-\hat{x})\delta(1-\hat{z}) \right. \right. \\
&\quad \left. \left. + \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1-\hat{x})_+(1-\hat{z})_+} + \frac{1+\hat{z}}{(1-\hat{x})_+} - 2(1-\hat{x}) + \delta(1-\hat{z}) \left( -(1-\hat{x})(1+2\hat{x}) \log \frac{\hat{x}}{1-\hat{x}} \right. \right. \right. \\
&\quad \left. \left. - 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ + \frac{2}{(1-\hat{x})_+} - 2(1-\hat{x}) + 2 \left( \frac{\ln \hat{x}}{(1-\hat{x})_+} \right) + \delta(1-\hat{x}) \left( (1+\hat{z}) \ln \hat{z} (1-\hat{z}) \right. \right. \right. \\
&\quad \left. \left. - 2 \left( \frac{\ln \hat{z}}{(1-\hat{z})_+} - 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{2\hat{z}}{(1-\hat{z})_+} \right) \right] + G_F^q(x, x_B) D^q(z) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ 2\delta(1-\hat{x})\delta(1-\hat{z}) \right. \right. \\
&\quad \left. \left. + \frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z}) \left( \log \frac{\hat{x}}{1-\hat{x}} + 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ - 2 \frac{\ln \hat{x}}{(1-\hat{x})_+} - \frac{1+\hat{x}}{(1-\hat{x})_+} \right) \right. \right. \\
&\quad \left. \left. + \delta(1-\hat{x}) \left( -(1+\hat{z}) \ln \hat{z} (1-\hat{z}) + 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + 2 \frac{\ln \hat{z}}{(1-\hat{z})_+} - \frac{2\hat{z}}{(1-\hat{z})_+} \right) \right] \right. \\
&\quad \left. + \tilde{G}_F^q(x, x_B) D^q(z) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ -\frac{1-\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 3 \right) \right] \right. \\
&\quad + G_F^q(x_B, x_B-x) D^q(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{(1-2\hat{x})\hat{z}^2}{(1-\hat{z})_+} - \delta(1-\hat{z})(1-2\hat{x}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 1 \right) \right) \right. \\
&\quad + \frac{1}{2\hat{z}} (1-2\hat{x}) \{ (1-\hat{z})^2 + \hat{z}^2 \} + \tilde{G}_F^q(x_B, x_B-x) D^q(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{\hat{z}^2}{(1-\hat{z})_+} \right. \right. \\
&\quad \left. \left. - \delta(1-\hat{z}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 3 \right) - \frac{1}{2\hat{z}} (1-2\hat{x}) \right] \right\} \left. \right], \tag{48}
\end{aligned}$$

where we performed partial integral,

$$\int_{x_B}^1 dx \frac{d}{dx} G_F(x, x) (1 + \hat{x}^2) = \int_{x_B}^1 \frac{dx}{x} G_F(x, x) (2\hat{x}^2 - 2\delta(1 - \hat{x})), \quad (49)$$

and we used  $G_F(x, x_B)\delta(1 - \hat{x}) = G_F(x, x)\delta(1 - \hat{x})$ . The boundary condition of the integrals is determined by the condition  $0 < P_{h\perp} = \sqrt{\frac{z^2(1-\hat{x})(1-\hat{z})}{\hat{x}}} Q^2 < P_{h\perp}^{\max}$ . The hard cross sections (45)-(47) associated with  $\tilde{G}(x, x_B)$ ,  $G(x_B, x_B - x)$  and  $\tilde{G}(x_B, x_B - x)$  are new results derived in this study and, in particular, the latter two contributions came from the HP2 contribution [27] which was not discussed in previous studies of the  $P_{h\perp}$ -weighted SSA. We should not neglect these new contributions to demonstrate the cancellation of the collinear singularities. Other contributions (42)-(44) agree with those derived in the previous study [26].

## 4 LO cross section and virtual-correction contribution in NLO cross section

In this section, we introduce the results of the LO cross section and virtual-correction contribution in NLO cross section already derived in [26]. Both contributions can be represented with  $2 \rightarrow 1$  scattering cross section. The phase-space integral should be changed from  $2 \rightarrow 2$  scattering as

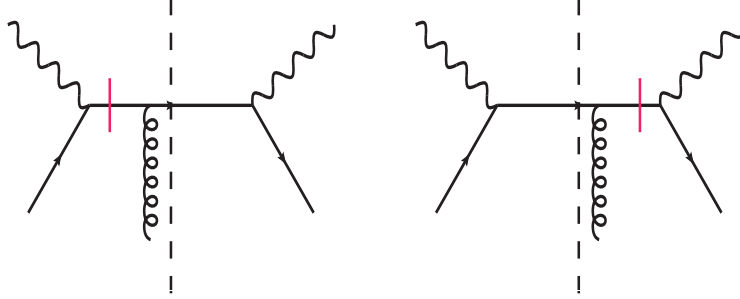


Figure 6: Leading order diagram for the  $P_{h\perp}$ -weighted cross section. Left(Right) figure represents  $S_{L(R)\gamma}(k_1, k_2)$ .

follows.

$$\begin{aligned} & \frac{d^3 p_c}{(2\pi)^3 2p_c^0} \frac{d^3 p_d}{(2\pi)^3 2p_d^0} (2\pi)^4 \delta^4(xp + q - p_c - p_d) \\ &= \frac{d^3 p_c}{(2\pi)^3 2p_c^0} (2\pi) \delta((xp + q - p_c)^2) \\ &\rightarrow \frac{d^3 p_c}{(2\pi)^3 2p_c^0} (2\pi)^4 \delta^4(xp + q - p_c) \end{aligned} \quad (50)$$

In this case, we perform  $P_{h\perp}$ -integration before the collinear expansion as

$$\int d^2 P_{h\perp} \epsilon^{\alpha\beta\gamma\delta} S_{\perp\alpha} P_{h\perp\beta} \left( S_{L\gamma}(k_1, k_2) \delta^2(k_{2\perp} - \frac{P_{h\perp}}{z}) + S_{R\gamma}(k_1, k_2) \delta^2(k_{1\perp} - \frac{P_{h\perp}}{z}) \right)$$

$$= \epsilon^{\alpha\beta pn} S_{\perp\alpha} z^3 \left( k_{2\perp\beta} S_{L\gamma}(k_1, k_2) + k_{1\perp\beta} S_{R\gamma}(k_1, k_2) \right), \quad (51)$$

where we used the fact the virtual photon doesn't have transverse momentum in hadron frame. We can find the following relation for LO diagram shown in Fig.6.

$$S_{Lp}(x_1p, x_2p) = -S_{Rp}(x_1p, x_2p) \equiv S_p(x_1p, x_2p). \quad (52)$$

Since the  $P_{h\perp}$ -integration brought  $O(k_\perp)$  term, the leading term of the collinear expansion gives twist-3 contribution. We can construct the gluon's field strength tensor as follows.

$$\begin{aligned} & \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i(k_2 - k_1) \cdot \eta} A^n(\eta) (k_{2\perp\beta} - k_{1\perp\beta}) S_p((k_1 \cdot n)p, (k_2 \cdot n)p) \\ &= i \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i(k_2 - k_1) \cdot \eta} F_\beta^n(\eta) S_p((k_1 \cdot n)p, (k_2 \cdot n)p) \\ &+ \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot \xi} e^{i(k_2 - k_1) \cdot \eta} A_\beta^\perp(\eta) (k_2 \cdot n - k_1 \cdot n) S_p((k_1 \cdot n)p, (k_2 \cdot n)p). \end{aligned} \quad (53)$$

The last term vanishes due to the SGP delta function. Then we can use the following formula for LO contribution

$$\begin{aligned} \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO}}}{dx_B dQ^2 dz_h d\phi} &= \frac{\alpha_{em}^2}{8z_h x_B^2 S_{ep}^2 Q^2} \int dz z D(z) \int dx_1 \int dx_2 \epsilon^{S_\perp \alpha p n} i M_{Fij\alpha}(x_1, x_2) \\ &\times \left( -g_{\mu\nu} H_{ijp}^{\mu\nu}(x_1p, x_2p) \right) \left( \frac{-2x\hat{x}}{\hat{u}Q^2} \right) \delta(x_1 - x_2) \delta(1 - \hat{x}) \delta(1 - \hat{z}), \end{aligned} \quad (54)$$

which agrees with the corresponding formula in [21, 26]. LO and NLO contributions in SIDIS were already calculated in previous work [26]. We just introduce their results in our notation below.

$$\frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 G^q(x_B, x_B) D^q(z_h) \quad (55)$$

$$\begin{aligned} \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{virtual}}}{dx_B dQ^2 dz_h d\phi} &= -\frac{z_h \pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \frac{\alpha_s}{2\pi} \sum_q e_q^2 G^q(x_B, x_B) D^q(z_h) \\ &\times \left[ C_F \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left( -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right) \right] \end{aligned} \quad (56)$$

Combining (48), (55) and (56), we obtain the following complete formula for NLO  $P_{h\perp}$ -weighted cross section.

$$\begin{aligned} & \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO+NLO}}}{dx_B dQ^2 dz_h d\phi} \\ &= -\frac{\pi z_h M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 \left[ G_F^q(x_B, x_B) D^q(z_h) + \frac{\alpha_s}{2\pi} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left( -\frac{1}{\epsilon} \right) \right] \end{aligned}$$

$$\begin{aligned}
& \times \left\{ D^q(z_h) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ P_{qq}(\hat{x}) G_F^q(x, x) + \frac{N}{2} \left( \frac{(1 + \hat{x}) G_F^q(x_B, x) - (1 + \hat{x}^2) G_F^q(x, x)}{(1 - \hat{x})_+} + \tilde{G}_F^q(x_B, x) \right) \right] \right. \right. \\
& - N G_F^q(x_B, x_B) + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1 - 2\hat{x}) G_F^q(x_B, x_B - x) + \tilde{G}_F^q(x_B, x_B - x) \right) \Big\} \\
& + G_F^q(x_B, x_B) \int_{z_h}^1 \frac{dz}{z} P_{qq}(\hat{z}) D^q(z) \Big\} \\
& + \frac{\alpha_s}{2\pi} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1 - \epsilon)} \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} \left\{ x \frac{dx}{x} G_F^q(x, x) D^q(z) \frac{1}{2N\hat{z}} \left[ 1 - \hat{z} + \frac{(1 - \hat{x})^2 + 2\hat{x}\hat{z}}{(1 - \hat{z})_+} \right. \right. \\
& - \delta(1 - \hat{z}) \left( (1 + \hat{x}^2) \ln \frac{\hat{x}}{1 - \hat{x}} + 2\hat{x} \right) \Big] + G_F^q(x, x) D^q(z) \frac{1}{2N\hat{z}} \left[ -2\delta(1 - \hat{x})\delta(1 - \hat{z}) \right. \\
& + \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})_+(1 - \hat{z})_+} + \frac{1 + \hat{z}}{(1 - \hat{x})_+} - 2(1 - \hat{x}) + \delta(1 - \hat{z}) \left( -(1 - \hat{x})(1 + 2\hat{x}) \log \frac{\hat{x}}{1 - \hat{x}} \right. \\
& - 2 \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \frac{2}{(1 - \hat{x})_+} - 2(1 - \hat{x}) + 2 \frac{\ln \hat{x}}{(1 - \hat{x})_+} \Big) + \delta(1 - \hat{x}) \left( (1 + \hat{z}) \ln \hat{z}(1 - \hat{z}) \right. \\
& - 2 \frac{\ln \hat{z}}{(1 - \hat{z})_+} - 2 \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + \frac{2\hat{z}}{(1 - \hat{z})_+} \Big] + G_F^q(x, x_B) D^q(z) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ 2\delta(1 - \hat{x})\delta(1 - \hat{z}) \right. \\
& + \frac{1 + \hat{x}\hat{z}^2}{(1 - \hat{x})_+(1 - \hat{z})_+} + \delta(1 - \hat{z}) \left( \log \frac{\hat{x}}{1 - \hat{x}} + 2 \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ - 2 \frac{\ln \hat{x}}{(1 - \hat{x})_+} - \frac{1 + \hat{x}}{(1 - \hat{x})_+} \right) \\
& + \delta(1 - \hat{x}) \left( -(1 + \hat{z}) \ln \hat{z}(1 - \hat{z}) + 2 \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + 2 \frac{\ln \hat{z}}{(1 - \hat{z})_+} - \frac{2\hat{z}}{(1 - \hat{z})_+} \right) \Big] \\
& + \tilde{G}_F^q(x, x_B) D^q(z) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ -\frac{1 - \hat{x}\hat{z}^2}{(1 - \hat{x})_+(1 - \hat{z})_+} + \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) \right] \\
& + G_F^q(x_B, x_B - x) D^q(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{(1 - 2\hat{x})\hat{z}^2}{(1 - \hat{z})_+} - \delta(1 - \hat{z})(1 - 2\hat{x}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 1 \right) \right) \right. \\
& + \frac{1}{2\hat{z}} (1 - 2\hat{x}) \{ (1 - \hat{z})^2 + \hat{z}^2 \} \Big] + \tilde{G}_F^q(x_B, x_B - x) D^q(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{\hat{z}^2}{(1 - \hat{z})_+} \right. \right. \\
& \left. \left. - \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) \right) - \frac{1}{2\hat{z}} (1 - 2\hat{x}) \right] - 8C_F \delta(1 - \hat{x})\delta(1 - \hat{z}) \Big\} \Big], \tag{57}
\end{aligned}$$

where  $P_{qq}(x)$  is the splitting function

$$P_{qq}(x) = C_F \left[ \frac{1 + x^2}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right]. \tag{58}$$

The double pole terms  $\frac{2}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z})$  are cancelled between the real cross section and the virtual cross section. The single pole term in virtual cross section  $2 \times \frac{3}{2}\delta(1-\hat{x})\delta(1-\hat{z})$  is incorporated into the splitting functions. The collinear singularities associated with the twist-3 functions can be subtracted with the following renormalization.

$$\begin{aligned}
& G_F(x_B, x_B) \\
&= G_F^{(0)}(x_B, x_B) + \frac{\alpha_s}{2\pi} \left( -\frac{1}{\hat{\epsilon}} \right) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ P_{qq}(\hat{x}) G_F(x, x) \right. \right. \\
&\quad \left. \left. + \frac{N}{2} \left( \frac{(1+\hat{x})G_F(x_B, x) - (1+\hat{x}^2)G_F(x, x)}{(1-\hat{x})_+} + \tilde{G}_F(x_B, x) \right) \right] - N G_F(x_B, x_B) \right. \\
&\quad \left. + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1-2\hat{x})G_F(x_B, x_B-x) + \tilde{G}_F(x_B, x_B-x) \right) \right\}, \tag{59}
\end{aligned}$$

where we adopted the  $\overline{\text{MS}}$ -scheme

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi. \tag{60}$$

These collinear singularities are the same as those in F-type correlator (1) at 1-loop order [22, 24, 25]. Then the collinear singularities are consistently subtracted and we can obtain the infrared-safe NLO cross section as follows.

$$\begin{aligned}
& \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO+NLO}}}{dx_B dQ^2 dz_h d\phi} \\
&= -\frac{\pi z_h M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 \left[ G_F^q(x_B, x_B, \mu) D^q(z_h, \mu) \right. \\
&\quad \left. + \frac{\alpha_s}{2\pi} \ln \left( \frac{Q^2}{\mu^2} \right) \left\{ D^q(z_h, \mu) \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ P_{qq}(\hat{x}) G_F^q(x, x, \mu) \right. \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{N}{2} \left( \frac{(1+\hat{x})G_F^q(x_B, x, \mu) - (1+\hat{x}^2)G_F^q(x, x, \mu)}{(1-\hat{x})_+} + \tilde{G}_F^q(x_B, x, \mu) \right) \right] \right. \right. \\
&\quad \left. \left. - N G_F^q(x_B, x_B, \mu) + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1-2\hat{x})G_F^q(x_B, x_B-x, \mu) + \tilde{G}_F^q(x_B, x_B-x, \mu) \right) \right\} \right. \\
&\quad \left. + G_F^q(x_B, x_B, \mu) \int_{z_h}^1 \frac{dz}{z} P_{qq}(\hat{z}) D^q(z, \mu) \right\} \\
&\quad \left. + \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} \left\{ x \frac{dx}{x} G_F^q(x, x, \mu) D^q(z, \mu) \frac{1}{2N\hat{z}} \left[ 1 - \hat{z} + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} \right. \right. \right. \\
&\quad \left. \left. \left. - \delta(1-\hat{z}) \left( (1+\hat{x}^2) \ln \frac{\hat{x}}{1-\hat{x}} + 2\hat{x} \right) \right] + G_F^q(x, x, \mu) D^q(z, \mu) \frac{1}{2N\hat{z}} \left[ -2\delta(1-\hat{x})\delta(1-\hat{z}) \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& + \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1-\hat{x})_+(1-\hat{z})_+} + \frac{1+\hat{z}}{(1-\hat{x})_+} - 2(1-\hat{x}) + \delta(1-\hat{z}) \left( -(1-\hat{x})(1+2\hat{x}) \log \frac{\hat{x}}{1-\hat{x}} \right. \\
& - 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ + \frac{2}{(1-\hat{x})_+} - 2(1-\hat{x}) + 2 \frac{\ln \hat{x}}{(1-\hat{x})_+} \Big) + \delta(1-\hat{x}) \left( (1+\hat{z}) \ln \hat{z}(1-\hat{z}) \right. \\
& \left. - 2 \frac{\ln \hat{z}}{(1-\hat{z})_+} - 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{2\hat{z}}{(1-\hat{z})_+} \right) \Big] \\
& + G_F^q(x, x_B, \mu) D^q(z, \mu) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ 2\delta(1-\hat{x})\delta(1-\hat{z}) + \frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} \right. \\
& + \delta(1-\hat{z}) \left( \log \frac{\hat{x}}{1-\hat{x}} + 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ - 2 \frac{\ln \hat{x}}{(1-\hat{x})_+} - \frac{1+\hat{x}}{(1-\hat{x})_+} \right) \\
& + \delta(1-\hat{x}) \left( -(1+\hat{z}) \ln \hat{z}(1-\hat{z}) + 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + 2 \frac{\ln \hat{z}}{(1-\hat{z})_+} - \frac{2\hat{z}}{(1-\hat{z})_+} \right) \Big] \\
& + \tilde{G}_F^q(x, x_B, \mu) D^q(z, \mu) \left( C_F + \frac{1}{2N\hat{z}} \right) \left[ -\frac{1-\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 3 \right) \right] \\
& + G_F^q(x_B, x_B - x, \mu) D^q(z, \mu) \left[ \frac{1}{2N\hat{z}} \left( \frac{(1-2\hat{x})\hat{z}^2}{(1-\hat{z})_+} - \delta(1-\hat{z})(1-2\hat{x}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 1 \right) \right) \right. \\
& + \frac{1}{2\hat{z}} (1-2\hat{x}) \{ (1-\hat{z})^2 + \hat{z}^2 \} \Big] + \tilde{G}_F^q(x_B, x_B - x, \mu) D^q(z, \mu) \left[ \frac{1}{2N\hat{z}} \left( \frac{\hat{z}^2}{(1-\hat{z})_+} \right. \right. \\
& \left. \left. - \delta(1-\hat{z}) \left( \ln \frac{\hat{x}}{1-\hat{x}} + 3 \right) \right) - \frac{1}{2\hat{z}} (1-2\hat{x}) \right] - 8C_F \delta(1-\hat{x})\delta(1-\hat{z}) \Big] + O(\alpha_s^2), \tag{61}
\end{aligned}$$

where the scale dependence of  $G_F(x, x, \mu^2)$  was introduced so that the cross section doesn't depend on the artificial scale  $\mu$ . Then we can derive the scale evolution equation of  $G_F(x, x, \mu^2)$  as

$$\begin{aligned}
& \frac{\partial}{\partial \ln \mu^2} \frac{d^4 \langle P_{h\perp} \Delta \sigma \rangle^{\text{LO+NLO}}}{dx_B dQ^2 dz_h d\phi} = 0 \\
& \rightarrow \frac{\partial}{\partial \ln \mu^2} G_F(x_B, x_B, \mu^2) = \frac{\alpha_s}{2\pi} \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ P_{qq}(\hat{x}) G_F(x, x, \mu^2) \right. \right. \\
& + \frac{N}{2} \left( \frac{(1+\hat{x}) G_F(x_B, x, \mu^2) - (1+\hat{x}^2) G_F(x, x, \mu^2)}{(1-\hat{x})_+} + \tilde{G}_F(x_B, x, \mu^2) \right) \Big] - N G_F(x_B, x_B, \mu^2) \\
& + \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1-2\hat{x}) G_F(x_B, x_B - x, \mu^2) + \tilde{G}_F(x_B, x_B - x, \mu^2) \right) \Big\} + O(\alpha_s^2), \tag{62}
\end{aligned}$$

which completely agrees with the results in [22, 24, 25].

## 5 Summary

We added the new hard pole contribution to the  $P_{h\perp}$ -weighted single-spin asymmetry in semi-inclusive deep inelastic scattering. Since the new pole contribution brings some collinear singularities at one-loop order, we should not neglect it for the exact cancellation of the collinear singularities. Our result showed that the NLO  $P_{h\perp}$ -weighted cross section has the same collinear singularities with the F-type correlator at one-loop order and then the singularities can be subtracted consistently. In addition, our calculation provided the scale evolution equation of the Qiu-Sterman function which completely agrees with the corresponding results in different approaches.

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